4.

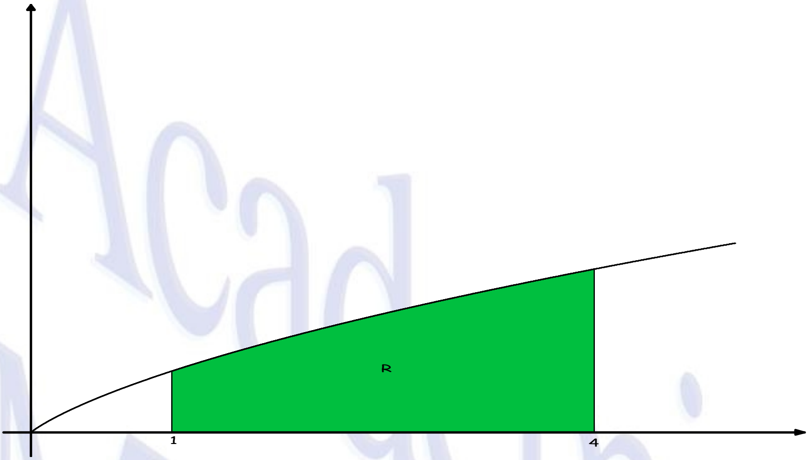

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y=\frac{x}{1+\sqrt{x}}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the line with equation x = 1 and the line with equation x = 4.

(a) Complete the table with the value of y corresponding to x = 3, giving your answer to 4 decimal places.

(1)

x	1	2	3	4
y	0.5	0.8284		1.3333

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate of the area of the region R, giving your answer to 3 decimal places.

(3)

(c) Use the substitution $u=1+\sqrt{x}$, to find, by integrating, the exact area of R.

(8)

a.

$$f(3)=\frac{3}{1+\sqrt{3}}\approx 1.0981$$

x	1	2	3	4
y	0.5	0.8284	1.0981	1.3333

b.

$$\int_1^4 \frac{x}{1+\sqrt{x}}dx \approx \frac{1}{2}\cdot h\cdot\left[f(1)+f(4)+2\cdot(f(2)+f(3))\right]=$$

$$=\frac{1}{2}\cdot 1\cdot[0.5+1.3333+2\cdot(0.8284+1.0981)]=2.84315\approx 2.843 \quad u^2$$

c.

$$u=1+\sqrt{x} \quad \Rightarrow \begin{cases} du=\frac{1}{2\sqrt{x}}dx \quad \Rightarrow \quad 2(u-1)du=dx \\ (u-1)^2=x \quad \Rightarrow \quad \text{if } x=1 \Rightarrow u=2 \quad \text{if } x=4 \Rightarrow u=3 \end{cases}$$

$$\int_1^4 \frac{x}{1+\sqrt{x}}dx=\int_2^3 \frac{2(u-1)^3}{u}du=\int_2^3\left(2u^2-6u+6-\frac{2}{u}\right)du=\left[\frac{2}{3}u^3-3u^2+6u-2\cdot\ln(u)\right]_2^3=$$

$$=\frac{54}{3}-\frac{16}{3}-27+12+18-12+2\cdot\ln\left(\frac{2}{3}\right)=\frac{11}{3}+2\cdot\ln\left(\frac{2}{3}\right) \quad u^2$$